John Leedham and D. V. Parker

Area and Volume

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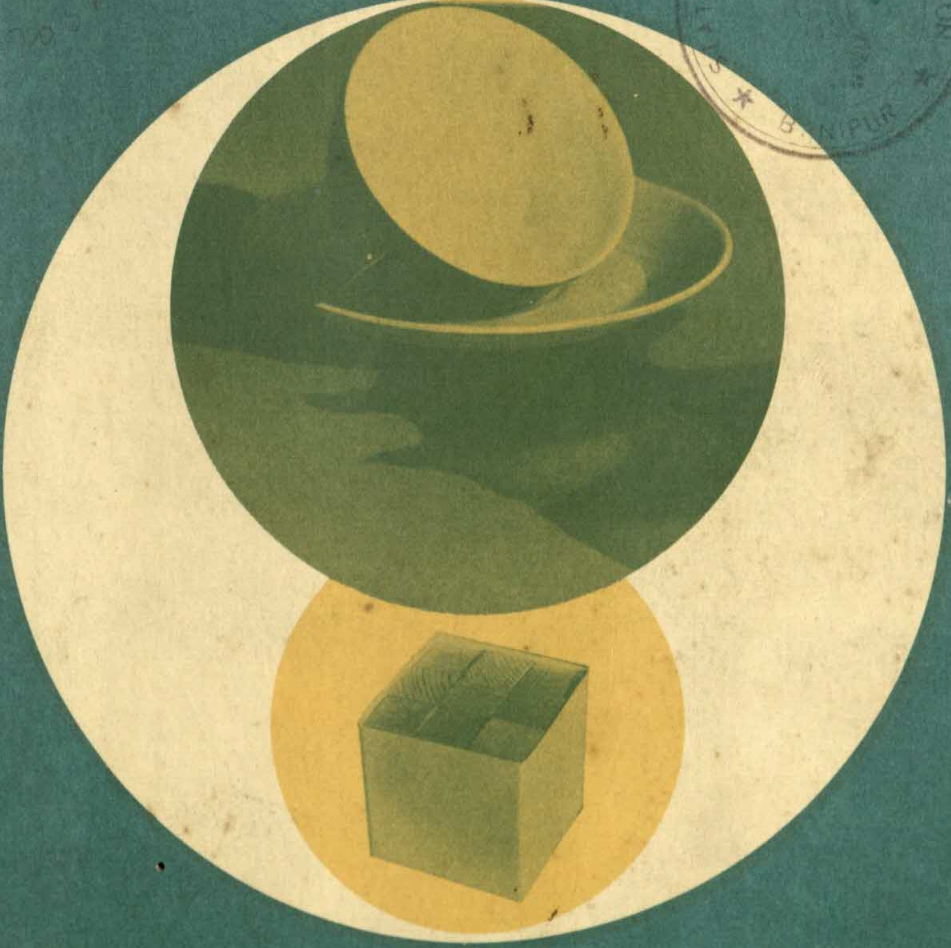

DISCOVERY PROGRAMMES

Area and Volume

This programme has the following test record:

Area

Tested with 124 children aged 10-13 in four localities
Error rate on all frames=14·7%
Post test score=56%
Further tested with 43 children in two localities
Error rate on all frames=7%
Post test score 63%
Error rate on published programme with representative population=6%
Post test score=71%

Volume

Validation tests and results were similar to those given for Area
Post test score=63%
Retention test for both programmes, given four weeks later, revealed a retention figure of 58%+

DISCOVERY PROGRAMMES
General Editor: John Leedham, M.Ed.

2051/5105

Area and volume

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Titles in this series will include

THE AIR WE BREATHE by John Leedham
A FIRST BOOK OF SETS
A SECOND BOOK OF SETS } by John Clarke
THE GEOMETRY OF THE POINT AND LINE
by Cyril Harries
THE SCARLET RUNNER by John Fieldhouse
DON'T SMOKE! by R. W. Kind and John Leedham
THE WORLD OF NUMBER by Norman Beard
A FIRST BOOK OF FRACTIONS
A SECOND BOOK OF FRACTIONS } by John Clarke
UNDERSTANDING BASE by John Clarke

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Introducing Discovery Programmes

Discovery Programmes are programmed books developed by practising teachers. They seek to introduce fresh material, or to put forward traditional material in a new way; they are intended to be a real adjunct to the work of the teacher. All have been carefully tested and have proved their worth under classroom conditions. The texts require a reading age of 10·4 or above; no previous knowledge of their subjects is needed.

Discovery Programmes are the fruit of experiments in programmed learning carried out by a group of teachers in the Leicestershire Education Authority. Our programmes were tested and re-tested in many schools in the locality, and the editor would like to express his appreciation of the help provided by the Director and Committee of the Leicestershire Education Authority.

"Programmed Learning is one of the most important new tools teachers have at their disposal. We are only at the beginning of exploring its full potential and I certainly will not attempt to guess how far it will take us. The important thing is to realise that it is a tool to be used by the teacher and not a substitute for him. A programme has the merit of infinite patience and it gives its undivided attention to the child who is using it. If it is a good one, it is subtle in providing the right doses of motivation."

Stewart C. Mason, M.A.
Director of Education, Leicestershire

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To the Teacher

This programme is for children in the top classes of the Junior School and the lower forms of the Secondary School. A reading age of 10 years or more is required. It is assumed that the pupil is able to measure and draw simple squares and rectangles.

The programme on AREA comprises:

Experiments about area
The concept of area
The concept of area measurement
Application of the square inch to the measurement of area
Finding the area of simple regular shapes
Revisionary Tests 1 and 2

A certain amount of simple apparatus is required. The following are usually available in the classroom:

a notebook and pencil, a ruler, scissors, tracing paper, squared arithmetic paper, a blackboard, and a damp cloth.

The pupil can usually provide:

a penny, a small torch, a small mirror.

The teacher may need to supply:

thick paper or thin card 6" square, cardboard or paper (for making a mask—see page 6).

The programme on VOLUME which follows that on AREA is devised to introduce the concept of volume rather than to give practice in computing it. The programme on VOLUME comprises:

Experiments in volume
Revision of area concept
The concept of volume
The concept of capacity

Measurement of volume by liquid measure
by cubic inch

Application of cubic measurement to regular solids

So that the pupils can perform the experiments which precede the programme on VOLUME, the teacher will need to provide:

a balloon, a bucket, a cup, a school milk bottle, and some sand.

At frame 44 in the VOLUME programme, the pupils may need the explanation that:

$1 \times 1 \times 1 = 1$
$2 \times 1 \times 1 = 2$
$2 \times 2 \times 1 = 4$
$2 \times 2 \times 2 = 8$
etc.

The pupil will require a simple mask (a piece of paper as wide and as long as this page) to cover the answers.

To the Pupil

This is a programme. This is how it works:

1 Cut a piece of cardboard or paper just long enough and wide enough to cover this page from top to bottom and side to side, including the coloured margin.
This we call a *mask*.
Place it over the frame below this one.
Now write in your notebook the answer to this question.
Is the coloured margin on the right—
$1\frac{1}{2}$ miles wide? $1\frac{1}{2}$ inches wide?
To find out if you are right, pull down the mask until you can just see the answer printed in the margin.

2 Mark your answer. Your answer should always be written down before you check it.
This is because we *LE – – N* better if we try on our own.
Write down the word.
Move down the mask.

1
$1\frac{1}{2}$ INCHES

3 Check your answer.
Now you know how to use the programme.

2
LEARN

AREA

Before you read and answer any question, be sure that the next frames are covered by your mask

AREA

First you need a torch and a penny.

Find a patch of wall in a dark or shady place. Shine your torch on to the wall. Now hold your penny between the torch and the wall and move it backwards and forwards until you can see the shape of its shadow clearly on the wall.
Now answer these questions.

1 Does the shape of the shadow cover any part of the wall?

ANSWER

---

2 Does the shape of the shadow stick out from the wall?

1
YES

Now make the shape of the shadow bigger.
Answer these questions.

3 Does the shape of the shadow cover more of the wall than before?

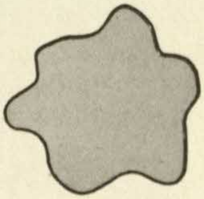
2
NO

---

4 Does the shadow stick out from the wall?

3
YES

Use a piece of paper and a pencil.
Draw and shade on your paper a shape like the one here.



5 Does the shape cover some of the surface of the paper?

4
NO

---

6 Does some of the shape stick out from the paper?

5
YES

You need a torch and a piece of card or thick paper with a hole cut in it.

Shine the torch on to a flat shady wall. Hold your card in front of the torch, and move it backwards and forwards until you can see the shape of your hole on the wall. Now move your torch (or your card) about so that the shape on the wall gets bigger or smaller.

7 As the shape gets bigger, does it cover more of the wall?

6 NO

---

8 As the shape gets bigger, does it stick out from the wall?

7 YES

---

9 Find a damp cloth and wipe a part of the blackboard with it. You can see a darker patch on the blackboard.

The darker patch c – – – – – – part of the blackboard.
Write out the missing word.

8 NO

Find a mirror, or a piece of flat glass.
Put your mouth close to the mirror or glass and breathe on it. You will see a misty patch appear on the mirror.
Stop breathing on the mirror and watch the patch.

10 After a few minutes does the patch cover more of the mirror, or does it cover less?

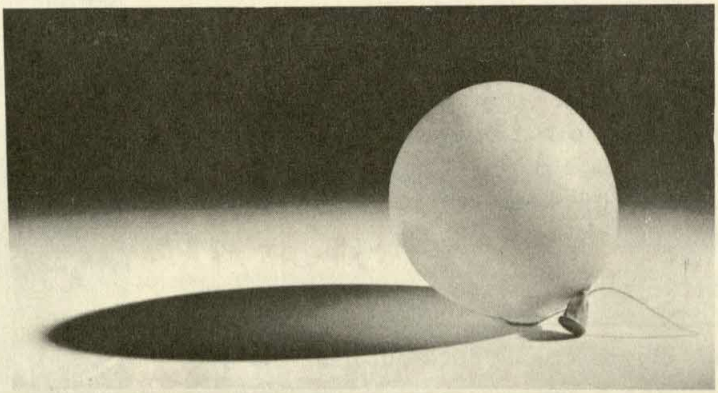
9 COVERS

---

11 Does the misty patch stick out from the mirror?

10 LESS

See if you can think of any other shapes like the ones you have just made. Remember they are shapes that cover something but do not stick out from it.



12 Look at this picture.
In the picture there is a shape like those you have been making. It *covers*, but does not *stick out*.
Which one is it—
(*a*) the balloon? (*b*) the string? (*c*) the shadow?

11
NO

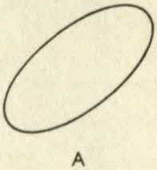
---

13 The shapes you made were all flat. Look at the picture again.
In the picture there is something that is not flat.
Is it—
(*a*) the balloon? (*b*) the shadow?

12
(*c*) THE SHADOW

---

14 A bucket is not flat; it can be filled with water. A bus is not flat; it can be filled with people. A box is not flat, it can be filled with many things. We can measure the space in these shapes. We say we are measuring their VOLUME. Which one of these shapes has VOLUME? (Which of them could be filled?)



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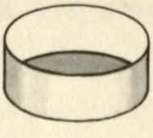

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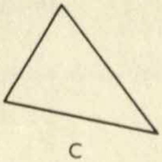

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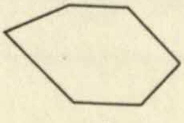
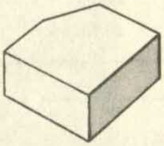


D

13
(*a*) THE BALLOON

15 A box, a bucket or a bus can be filled. Shapes which are flat cannot be filled. They have no volume.
Shapes which are flat cannot be filled, but they can be covered. We say that shapes which can be covered, but not filled, have AREA.

Which of these shapes has AREA but not VOLUME ?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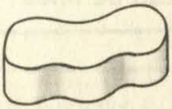
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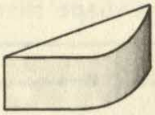
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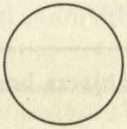
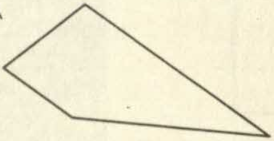

D

14
B

---

16 Which of these shapes has VOLUME ?

A


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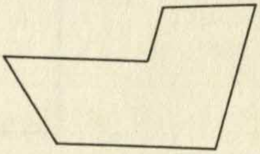

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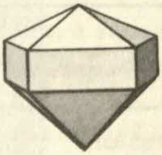

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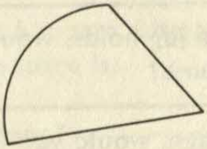
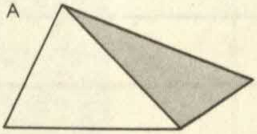


15
D

---

17 Which of these shapes has only AREA ?

A


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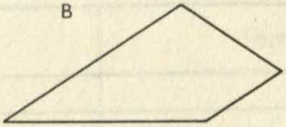

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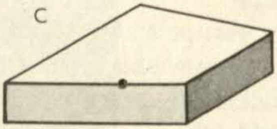

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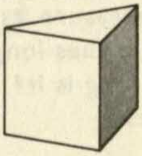


16
C

18	If a shape can be filled we say it has v – – – – – . Write the missing word in your notebook.	17 B
19	A shape that can be filled has VOLUME. A shape that can only be covered has – – – – . Write the missing word.	18 VOLUME
20	If a shape has only AREA, is it a shape that can be filled?	19 AREA
21	Which of these objects has volume— a shadow? a house? a patch of light?	20 NO
22	Which of these objects has only area— a loaf of bread? a bucket? a box? a patch of light?	21 A HOUSE
23	If you measured what a jug holds, would you be measuring area or volume?	22 A PATCH OF LIGHT
24	If you measured this page, would you measure its area or its volume?	23 VOLUME
25	Does a shadow have volume?	24 AREA
26	Does a ball have volume?	25 NO
27	A ———————————————— B Here is a line AB. We can measure its length with a ruler. We say it is so many inches long. Measure the line AB. How long is it? (*a*) 1 inch (*c*) 3 inches (*b*) 3 (*d*) 3 feet	26 YES

28 When we measure the LENGTH of a line we measure it in INCHES.

When we measure the AREA of a space we cannot measure it in ordinary inches.

Which of these can we measure in ordinary inches?

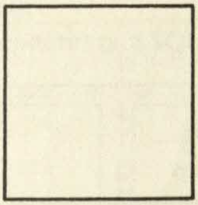
(*a*) a line
(*b*) an area
(*c*) volume
(*d*) weight

27
(*c*) 3 INCHES

29 We use SQUARE INCHES to measure AREA.

Now we must find out what is meant by 'a square inch'.

Here is one:

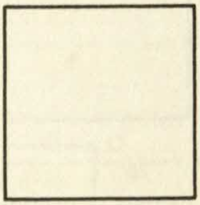


As you see, a square inch can be a square. Use a ruler and see how LONG one SIDE of the square is.

Is ONE side—

(*a*) 2 inches long?
(*b*) 4 inches long?
(*c*) 1 inch long?
(*d*) 1 square inch?

28
(*a*) A LINE



30 Here is a square.

Each side measures 1 inch.

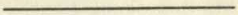
Is it a SQUARE INCH?

(*a*) Yes (*b*) No

29
(*c*) 1 INCH LONG

31 We measure length in inches and area in square inches. Which of these would we measure in SQUARE INCHES ?

A



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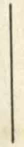

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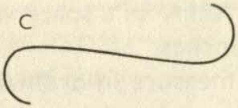

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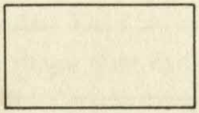
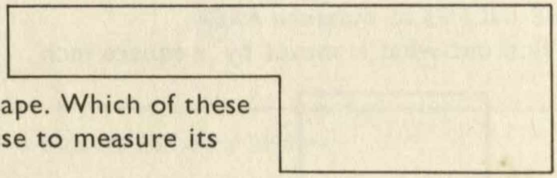


30
(*a*) YES



32 Here is a shape. Which of these would we use to measure its area?

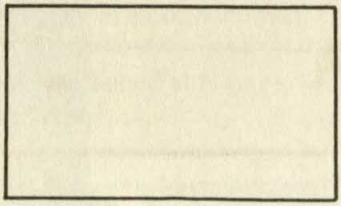
(*a*) inches
(*b*) square inches
(*c*) cubic inches
(*d*) round inches

31
D

---

33 Which of these is a square inch?

A



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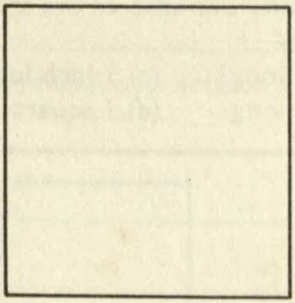

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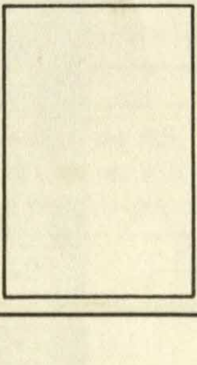

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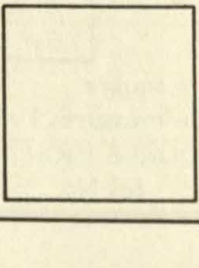


32
(*b*) SQUARE INCHES

2031

34 If we have a short line we measure it in inches.
If we have a longer line we measure it in feet.
If we have a small area we measure it in square inches.
If we have a bigger area we measure it in SQUARE FEET.
If we had a small cocoa tin lid, would we measure its area in—

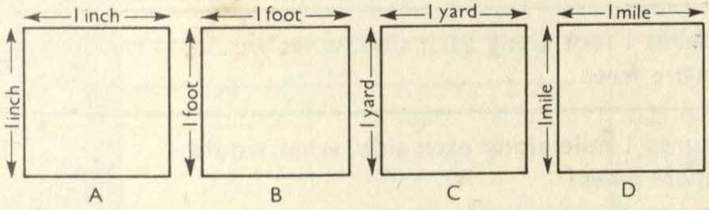
(*a*) square feet?
(*b*) square inches?

---

35 To measure long lines we use yards.
To measure large areas we use SQUARE YARDS.

A SQUARE YARD can be a square the sides of which each measures a yard.

Which of these represents a SQUARE YARD?



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36 We measure very big areas in SQUARE MILES.
A SQUARE MILE can be a square.
What would be the length of each side of the square?

(*a*) a mile
(*b*) a yard
(*c*) a foot
(*d*) an inch

---

37 If we use *inches* to measure length, then what sort of inches do we use to measure area?

---

38 Which is the bigger—a square inch or a square foot?

33
D

34
(*b*) SQUARE INCHES

35
C

36
(*a*) A MILE

37
SQUARE INCH

39 Which is the bigger, a square foot or a square mile?

---

40 Is a square mile bigger than a square yard?

---

41 If a square had an area of 1 square inch, how long would each side measure?

1 inch 2 inches 3 inches 4 inches

---

42 If a square had an area of 1 square foot, how long would one side be?

1 foot 1 square foot 1 square yard 1 square inch

---

43 If a square measures 1 inch along each side, what would its area be?

1 inch 1 square inch 1 square foot 1 square yard

---

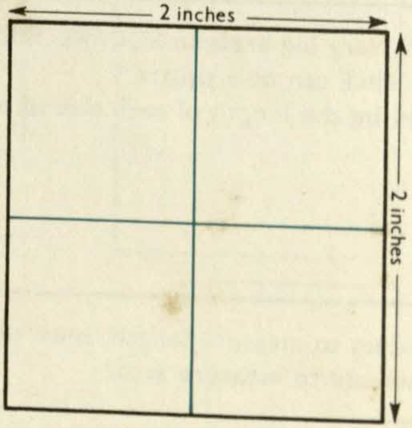
44 If a square measures 1 foot along each side, what would its area be *in square feet?*

---

45 If a square measures 1 mile along each side, what would its area be *in square miles?*

---

46 Here is a square.
It is two inches long and two inches wide.
It has been divided into small squares, each of which is a square inch.



How many small squares, each a square inch, does it take to make the big square which is 2 inches long and 2 inches wide?

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) 4

38
A SQUARE FOOT

39
A SQUARE MILE

40
YES

41
1 INCH

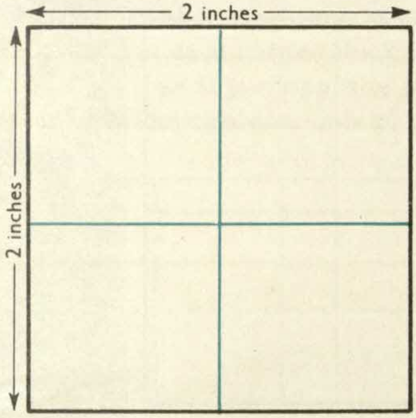
42
1 FOOT

43
1 SQUARE INCH

44
1 SQUARE FOOT

45
1 SQUARE MILE

47



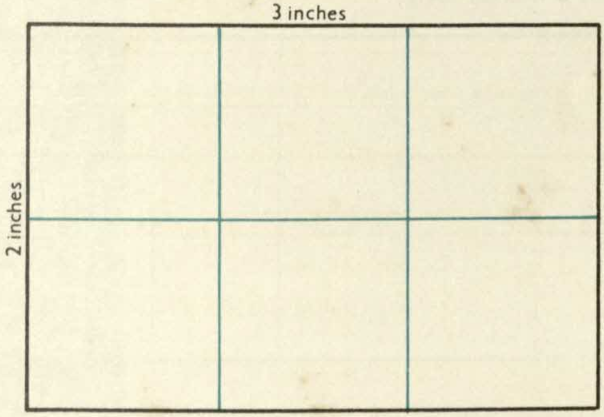
Here is the same square again.
You said it took 4 small squares, each a square inch, to make the big square.
Because of this we say the area of the big square is 4 square inches.

In what do we measure area?

(*a*) inches (*c*) cubic inches
(*b*) square inches (*d*) round inches

46
4

48



Here is a rectangle.
It has been divided into square inches for you.

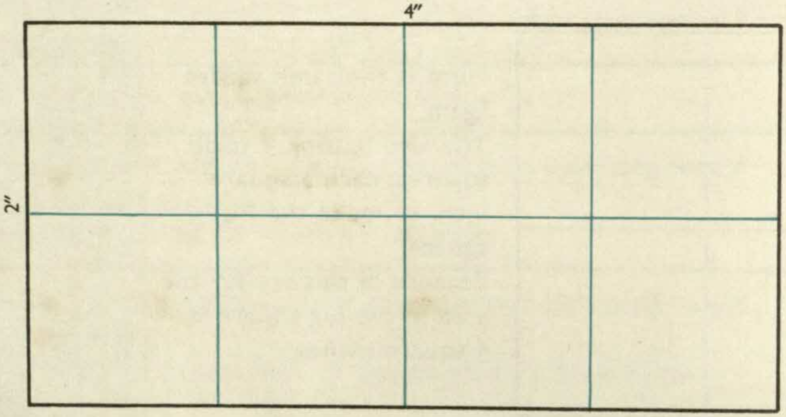
Remember, the area is the number of ALL the square inches it takes to make the rectangle. Count the square inches.

Is the area of the rectangle—

(*a*) 3 square inches? (*c*) 6 inches long?
(*b*) 2 square inches? (*d*) 6 square inches?

47
(*b*) SQUARE INCHES

49



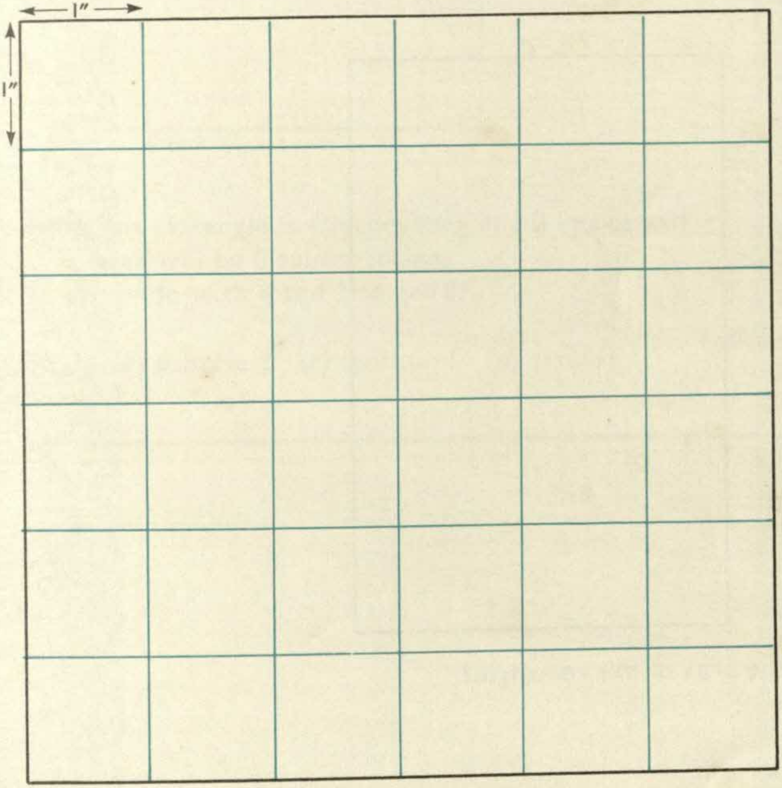
Here is another rectangle divided into square inches.
Remember, ″ stands for inch or inches.
What is the area of the rectangle?

(*a*) 4 square inches	(*c*) 8 square inches
(*b*) 2 square inches	(*d*) 8 inches long

48
(*d*) 6 SQUARE INCHES

49
(*c*) 8 SQUARE INCHES

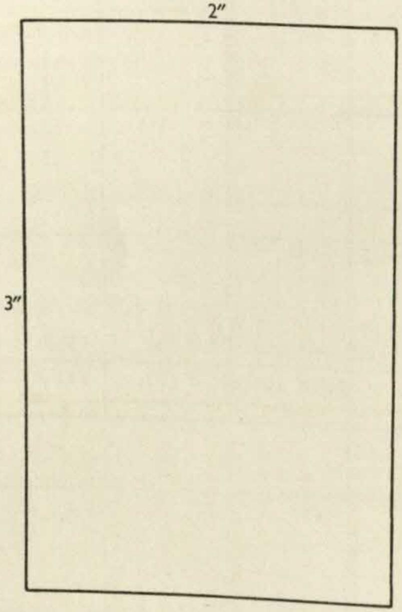
On a sheet of tracing paper 6 inches square, measure off inches from each corner along all four sides. Then with ruler and pencil join opposite points to make a grid. It will look like this one, only of course it will be bigger.



You now have a measuring grid. If you measure the squares, you will find each one is 1 inch long and 1 inch wide. So each square has an area of 1 square inch. You can use this grid to measure the areas of bigger shapes.

When your grid is ready, move on to frame 50.

50 Place the grid over the shape below and move it until the lines on the grid cover the lines at the edge of the shape. Now count all the squares it takes to cover the shape completely. This will be the area of the shape in square inches.



What is the area of this rectangle?

---

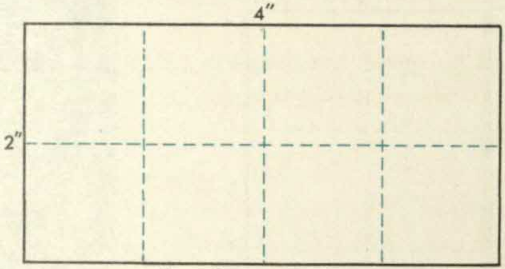
51 On squared arithmetic paper, draw these rectangles:

A 2 in. long and 3 in. wide;
B 3 in. long and 3 in. wide;
C 6 in. long and 2 in. wide;
D 5 in. long and 2 in. wide.

Find the area of each one by using your grid.

50
6 SQUARE INCHES

52

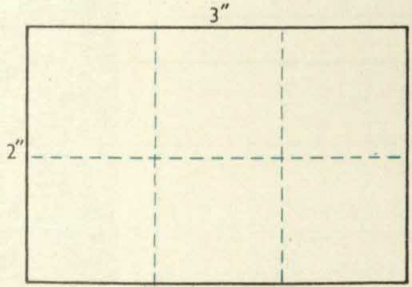


Area 8 square inches

Suppose this rectangle is 4 inches long and 2 inches wide.
Then its area will be 8 square inches.
What do we do with 4 and 2 to get 8?
Do we—
(*a*) add? (*b*) subtract? (*c*) multiply? (*d*) divide?

51
A= 6 SQ. IN.
B= 9 SQ. IN.
C=12 SQ. IN.
D=10 SQ. IN.

53



Area 6 square inches

Suppose this rectangle is 3 inches long and 2 inches wide.
Then its area will be 6 square inches.
What do we do with 3 and 2 to get 6?
Do we—
(*a*) add? (*b*) subtract? (*c*) multiply? (*d*) divide?

52
(c) MULTIPLY

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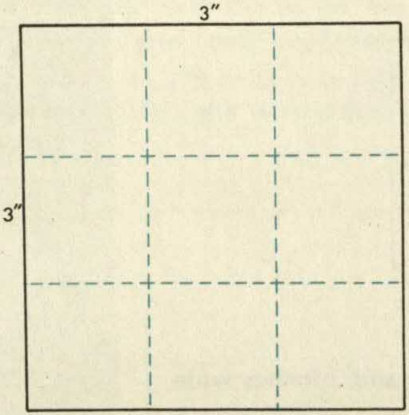
54



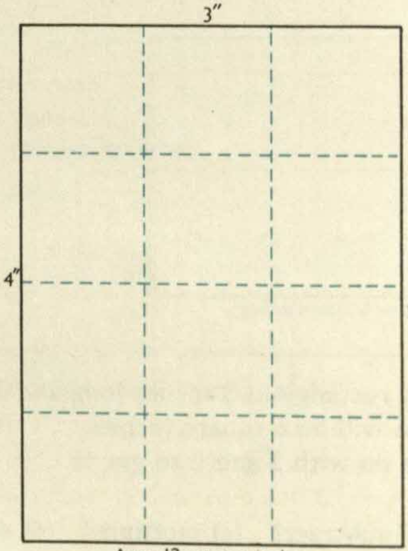
Area 9 square inches

Suppose the area of this rectangle is 9 square inches. What numbers should we multiply together to get the answer 9 square inches?

53
(*c*) MULTIPLY

55

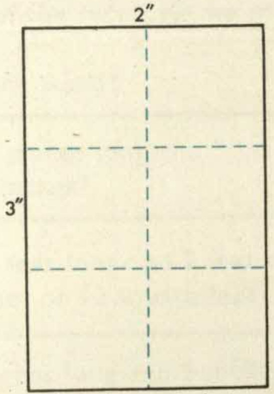
Suppose this rectangle is 3 inches long and 4 inches wide.
Then its area will be 12 square inches.
What do we do with 3 and 4 to make 12?
Do we—
(*a*) multiply?
(*b*) divide?
(*c*) add?
(*d*) subtract?



Area 12 square inches

54
3 × 3

56

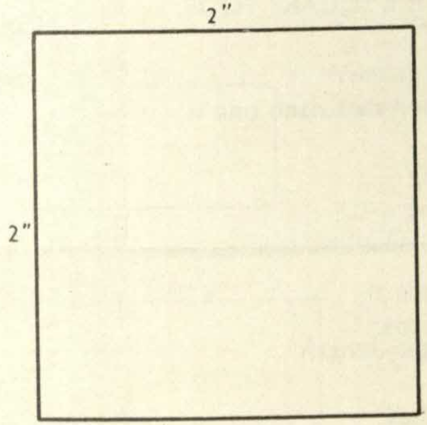


Area 6 square inches

Suppose this rectangle is 2 inches long and 3 inches wide.
Then its area will be 6 square inches.
What do we do with 2 and 3 to get 6?
Do we—
(*a*) add? (*b*) multiply? (*c*) subtract? (*d*) divide?

55
(*a*) MULTIPLY

57



(Remember, ″ means inch or inches.)

To find the area of this square we multiply the length by the width. This gives us the number of square inches.

What is the area of the square?

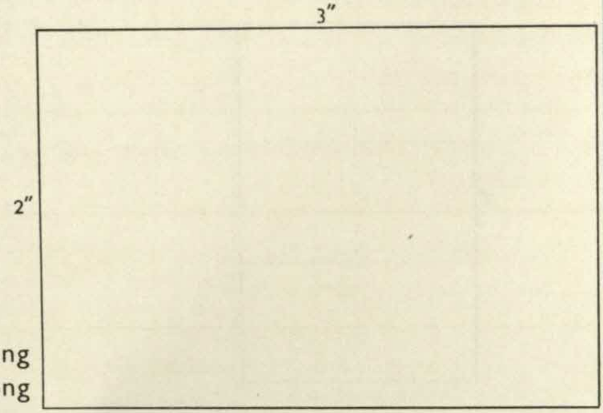
(*a*) 2 inches long (*c*) 4 inches long
(*b*) 2 square inches (*d*) 4 square inches

56
(*b*) MULTIPLY

58 Area=length
×width.

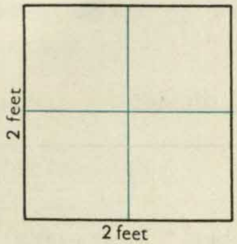
What is the
area of this
rectangle?

(*a*) 6 square inches
(*b*) 3 square inches
(*c*) 6 inches long
(*d*) 5 inches long



57
(*d*) 4 SQUARE INCHES

59



Here is a square. Suppose that it measures 2 feet long and 2 feet wide. It has been divided into smaller squares each of which is a SQUARE FOOT.

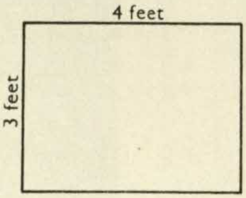
What is the area of the big square?
(Count the small squares and remember that each one is a SQUARE FOOT.)

(*a*) 4 square feet
(*b*) 4 square inches
(*c*) 4 inches long
(*d*) 2 square feet

58
(*a*) 6 SQUARE INCHES

---

60 Here is a rectangle. What is its area?



(Remember: Area=length ×width.)

(*a*) 12 square inches
(*b*) 12 square feet
(*c*) 7 square feet
(*d*) 7 square inches

59
(*a*) 4 SQUARE FEET

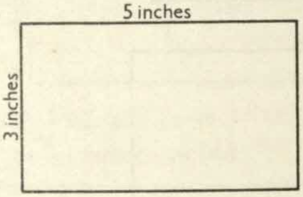
61 To find the area of the rectangle we multiply length by ------ .
What is the missing word?

62 If a rectangle is 2 inches long and 3 inches wide, will its area be 6 square inches?

63 If a rectangle is 6 feet long and 2 feet wide, will its area be 12 square inches or 12 square feet?

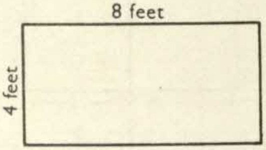
64 If a square is 4 inches long and 5 inches wide, what is its area?

65



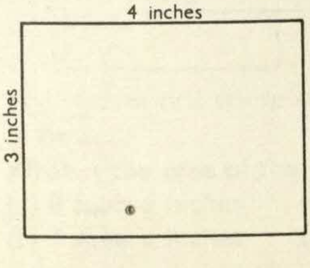
What is the area of this rectangle?

66



What is the area of this rectangle?

67



Is the area of this rectangle 12 square inches?

60
(*b*) 12 SQUARE FEET

61
WIDTH

62
YES

63
12 SQUARE FEET

64
20 SQUARE INCHES

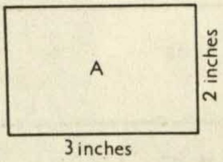
65
15 SQUARE INCHES

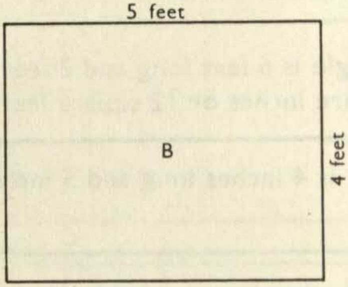
66
32 SQUARE FEET

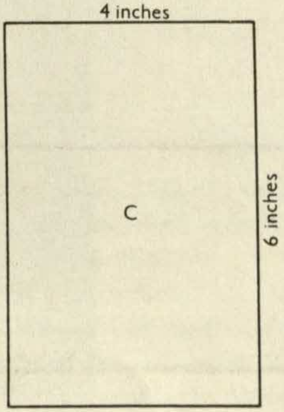
67
YES

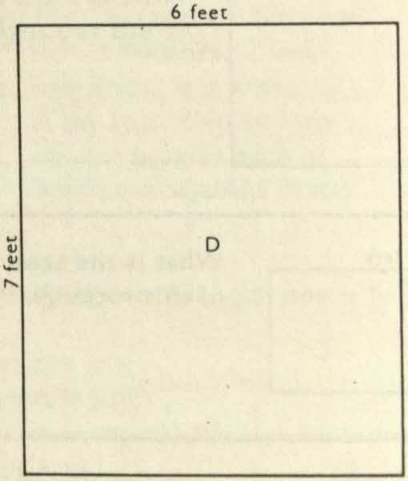
Test 1

Find the areas of these shapes, without measuring.



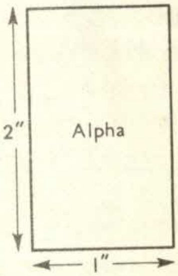


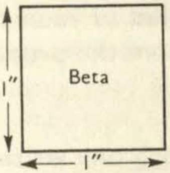




Take your answers to your teacher.

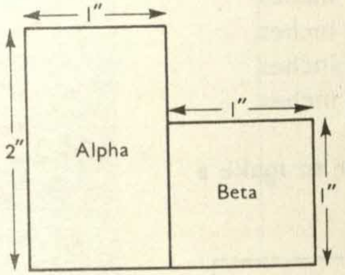
68





Here are two shapes.
We will call one
Alpha and the other
Beta.

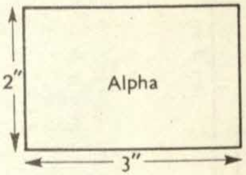
The area of Alpha is 2 square inches.
The area of Beta is 1 square inch.

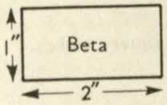


If we put Alpha and Beta together we get a shape like this. The area of the whole shape will be the area of Alpha + the area of Beta. What will the area of the whole shape be?

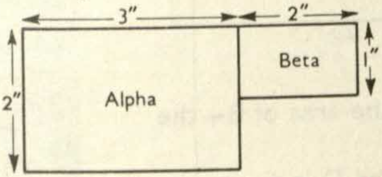
(*a*) 1 square inch
(*b*) 2 square inches
(*c*) 3 square inches
(*d*) 4 square inches

69





Here are two more shapes called Alpha and Beta.



If we put Alpha and Beta together we get a shape like this.

The area of this shape will be the area of Alpha + the area of Beta.

What is the area of the whole shape?

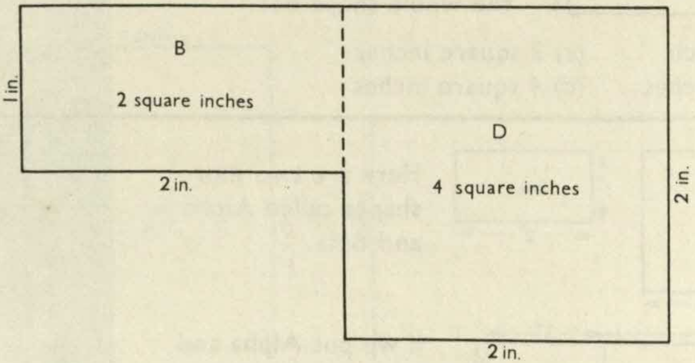
(*a*) 8 square inches
(*b*) 7 square inches
(*c*) 8 inches long
(*d*) 8 square feet

68
(c) 3 SQUARE INCHES

70 Make yourself some squares and rectangles. (If you use squared arithmetic paper you will get them the proper shape.)

Make six separate shapes A—F and on each one write its area.
Make A 1″ long and 1″ wide—area 1 square inch.
Make B 2″ long and 1″ wide—area 2 square inches.
Make C 3″ long and 1″ wide—area 3 square inches.
Make D 2″ long and 2″ wide—area 4 square inches.
Make E 3″ long and 2″ wide—area 6 square inches.
Make F 3″ long and 3″ wide—area 9 square inches.

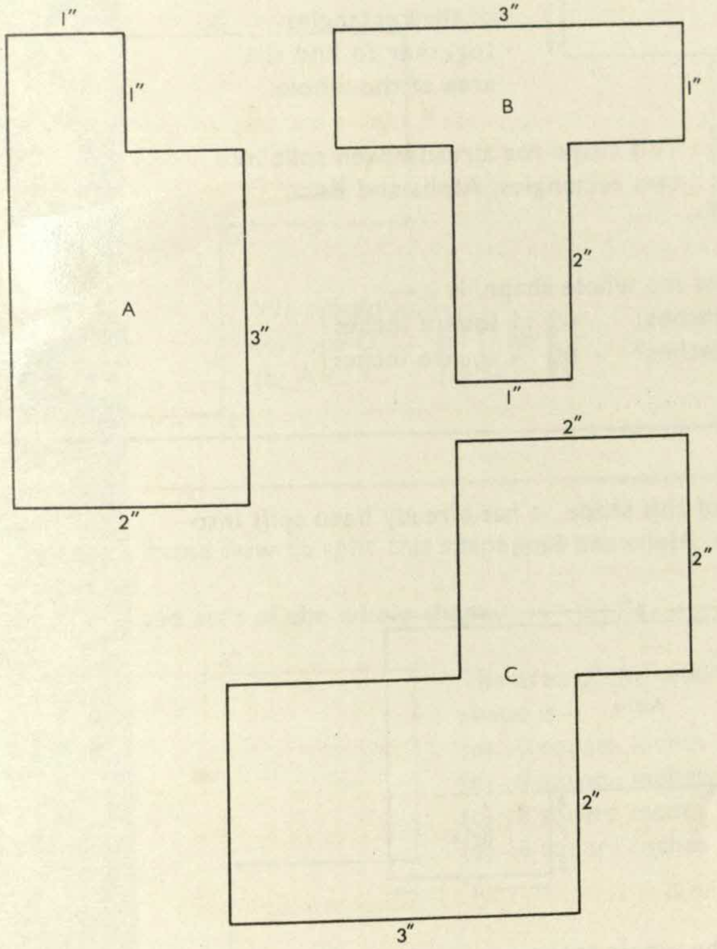
Take shapes B and D and put them together to make a figure like the one below.



The area of the whole shape will be the area of B+the area of D.
You know that B is 2 square inches and D is 4 square inches.
What is the area of the whole shape? Is it—
(*a*) 2 square inches?
(*b*) 3 square inches?
(*c*) 4 square inches?
(*d*) 6 square inches?

69
(*a*) 8 SQUARE INCHES

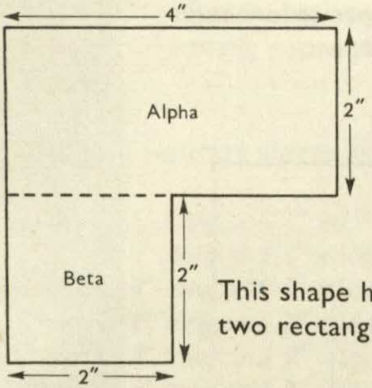
71 Use your shapes A—F to build up the shapes below and then work out the areas and write them down.
Check them by using your grid.



Try making up some more shapes of your own in the same way.

70
(*d*) 6 SQUARE INCHES

72



When we get a shape like this we must first split it into rectangles. We then add the areas of the rectangles together to find the area of the whole.

This shape has already been split into two rectangles, Alpha and Beta.

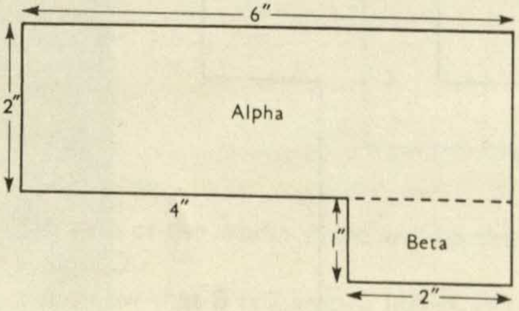
Find the area of the whole shape. Is it—

(*a*) 12 square inches? (*c*) 16 square inches?
(*b*) 8 square inches? (*d*) 4 square inches?

71
A= 7 SQ. IN.
B= 5 SQ. IN.
C=10 SQ. IN.

---

73 Find the area of this shape. It has already been split into two rectangles, Alpha and Beta.

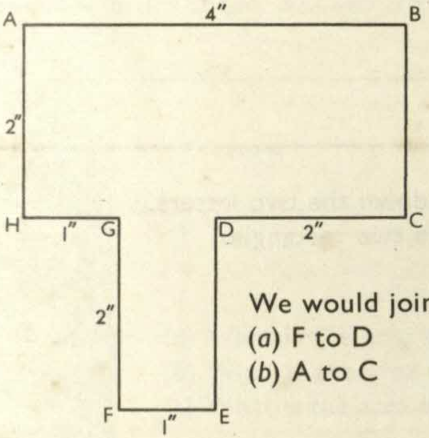


Is the area of the whole shape—

(*a*) 18 square inches? (*c*) 14 square inches?
(*b*) 8 square inches? (*d*) 20 square inches?

72
(*a*) 12 SQUARE INCHES

74 Here is a shape. To find its area we must first divide it into two rectangles.
Which two points would we join in order to split this shape into two rectangles?



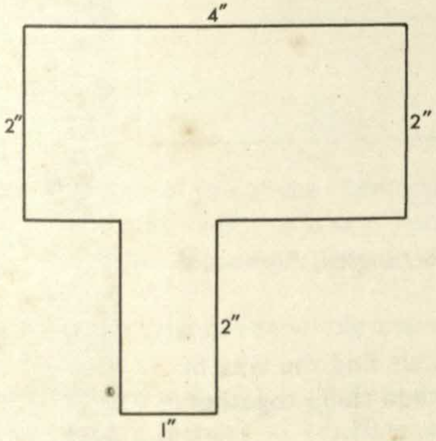
We would join—

(*a*) F to D
(*b*) A to C
(*c*) D to B
(*d*) G to D

73
(*c*) 14 SQUAR
INCHES

---

75 You have found how to split this shape into two rectangles.
Now find the area of the whole shape.

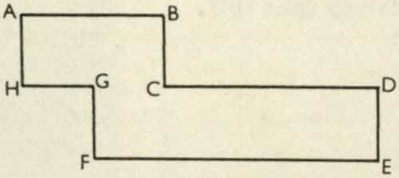


The area of the whole shape is—

(*a*) 10 square inches
(*b*) 9 square inches
(*c*) 12 square inches
(*d*) 16 square inches

74
(*d*) G TO D

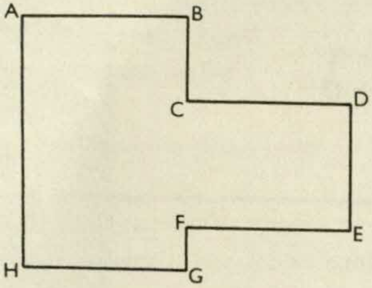
76 Which two points would you join in order to split this shape into two rectangles? Write down the two letters.



75
(*a*) 10 SQUARE INCHES

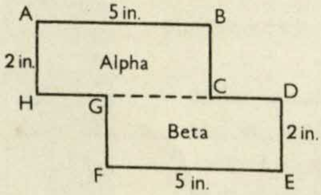
---

77 Do the same with this shape. Write down the two letters you would join to split the shape into two rectangles.



76
C AND G

78



This shape has been split into two rectangles, Alpha and Beta, by joining C to G.

To find the area of the whole shape, we find the area of Alpha and the area of Beta and then add them together. What is the area of shape Alpha?

77
C AND F

79 What is the area of shape Beta in frame 78?

78
10 SQ. IN.

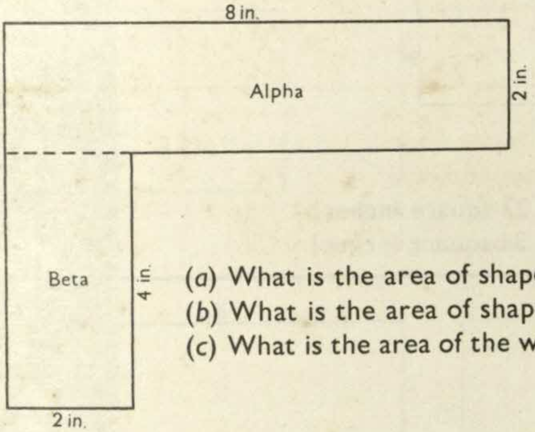
---

80 What is the area of the whole shape in frame 78?

79
10 SQ. IN.

---

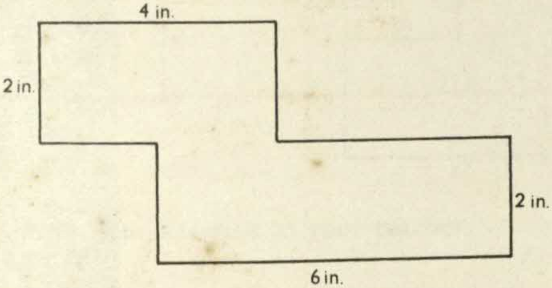
81 Here is another shape. Answer the questions in the same way.



(*a*) What is the area of shape Alpha?
(*b*) What is the area of shape Beta?
(*c*) What is the area of the whole shape?

80
20 SQ. IN.

82



Find the area of this shape. Remember first to split it into two rectangles. Find the area of each rectangle and add them together. This will give you the area of the whole shape.

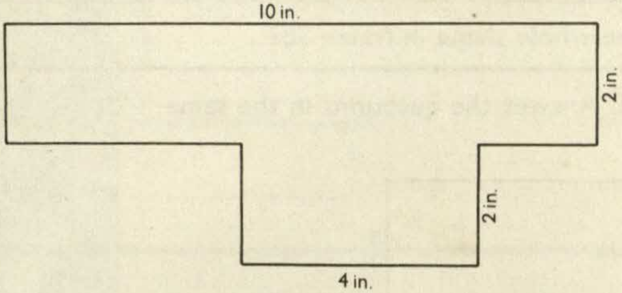
What is the area of the whole shape?

Is it—

(*a*) 10 square inches?
(*b*) 14 square inches?
(*c*) 24 square inches?
(*d*) 20 square inches?

81
(*a*) 16 SQ. IN.
(*b*) 8 SQ. IN.
(*c*) 24 SQ. IN.

83 Now find the area of this shape in the same way.



Is it—

(*a*) 28 square inches? (*c*) 22 square inches?

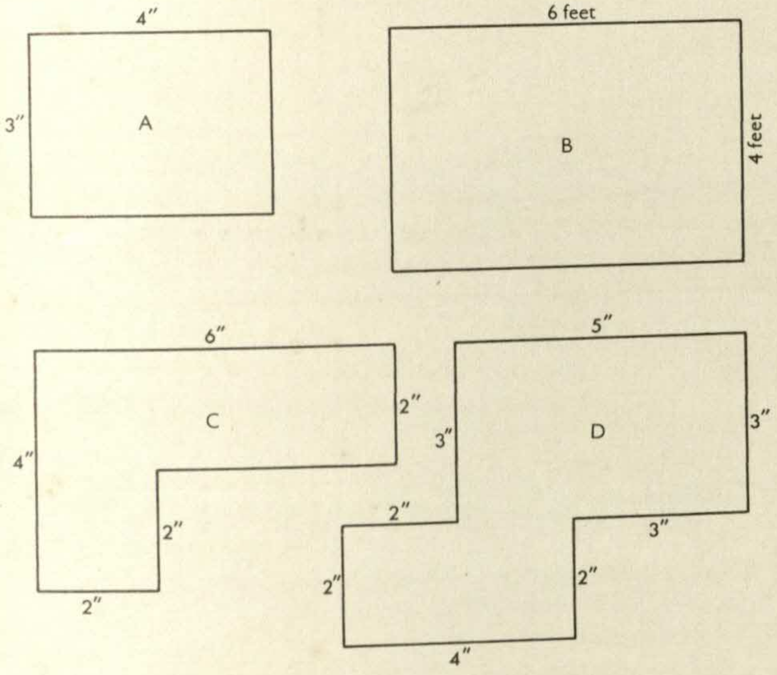
(*b*) 30 square inches? (*d*) 34 square inches?

82
(*d*) 20 SQ. IN.

83
(*a*) 28 SQ. IN.

Test 2

Find the areas of these shapes.



Take your answers to your teacher.

VOLUME

To the Pupil

Experiments

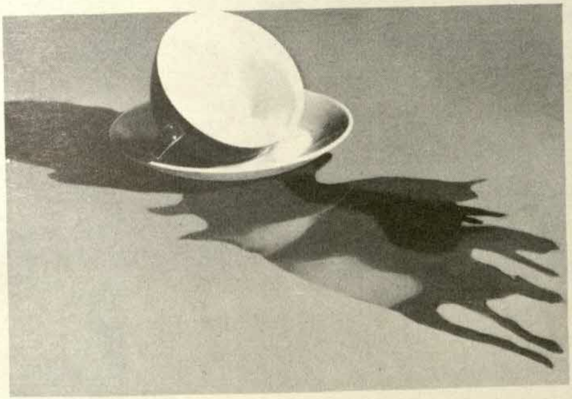
Before you begin the second part of this programme, try out these experiments.

1 Get a balloon, and a bucket full to the brim with water. Stand the bucket in a sink (or ask if you may take it outside). Blow the balloon half up, and push it carefully *under* the water without putting in your hand. Take out the balloon, and mark where the water now comes to on the side of the bucket. Fill up the bucket to the brim again. Blow the balloon right up and press it carefully under the water once more. Take it out and make another mark where the water is now.
What do the two marks show?

2 Get a cup full of water. Ask if you may take it out of doors. Throw the water into the air. Watch for it to land. Try again with the same amount of water. Does the water spread out in just the same way, or does it make a different pattern?

3 Get a school milk bottle. Fill it with sand, then tip out the sand on to a clean piece of paper. Fill the milk bottle with water. Look carefully at the water in the bottle and at the sand.
Do you think that the sand takes up as much space as the water?

Before you read and answer any question, be sure that the next frames are covered by your mask



1 This cup was FULL of water.
All the water is now spread on the SURFACE of the ground.
Was there more water—
(*a*) before it was spilt?
(*b*) after it was spilt?
(*c*) was the amount of water the same before and after it was spilt?

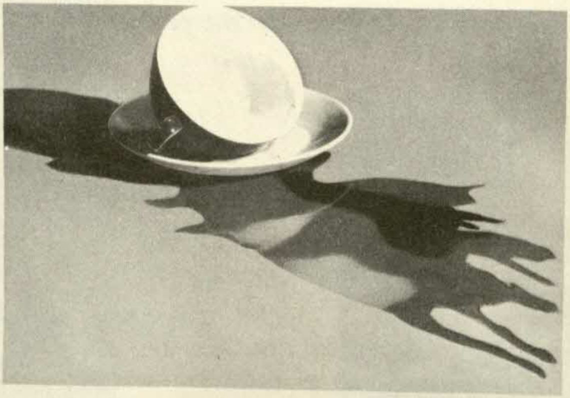
ANSWER

---

2 When the cup was full the water was all together.
When the cup was tipped up the water was spread out.

When did the water cover a greater AREA —
(*a*) in the cup? (*b*) on the ground?

1
(*c*) THE SAME BEFORE AND AFTER



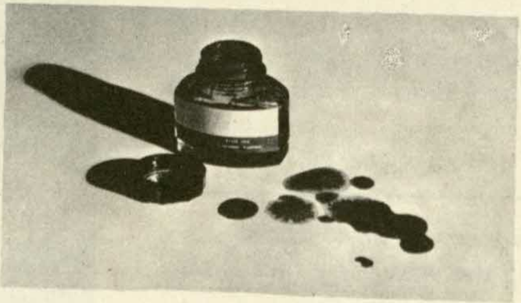
3 Look at the cup.
It could hold some water.
It has VOLUME.
Look at the reflection of the saucer.
It could not hold water.
It has only a – – – .

2
(*b*) ON THE GROUND

4 One difference between a thing having AREA and a thing having VOLUME is that one of them only covers a surface. Is this—

(*a*) the one having AREA? (*b*) the one having VOLUME?

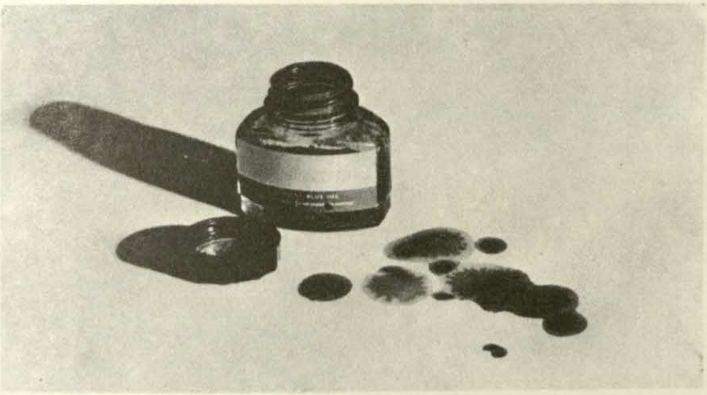
3
AREA



5 Which has volume—

the bottle? the blot? the shadow?

4
(*a*) THE ONE HAVING AREA



6 Which has volume—
the lid?
the shadow?

5
THE BOTTLE

7 Which has area (think hard!)—
the label?
the shadow?

6
THE LID

8 The bottle can be filled with ink.
It has v-----.

7
BOTH HAVE

9 In the bottle the ink stretched 3 ways:
from side to side;
from front to back;
from top to ------.

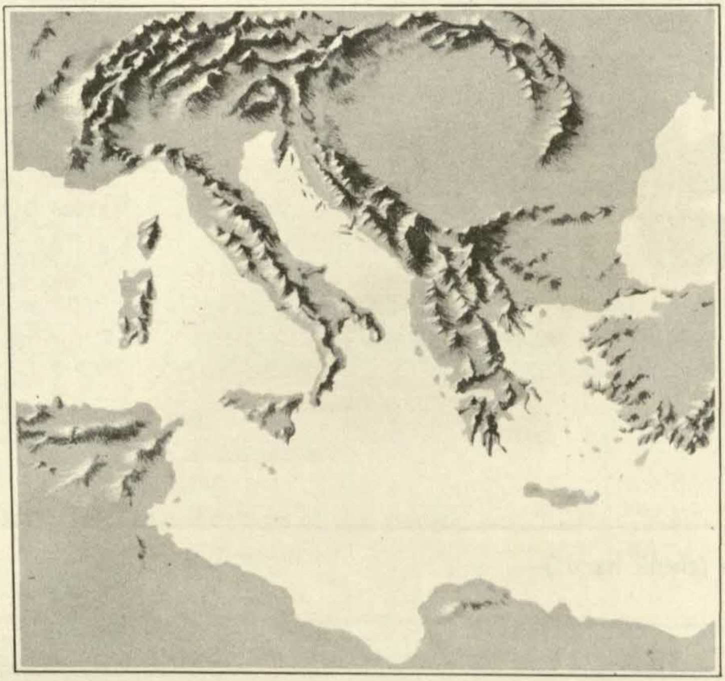
8
VOLUME

10 Look at the shadow. It stretches 2 ways.
It stretches from s--- -- s---
and from f---- -- b---.

9
BOTTOM

11 The bottle has v-----.
The shadow has a---.

10
SIDE TO SIDE
FRONT TO
BACK



12 Here is a different way of showing area.
This is a picture of a m – – .

11
VOLUME
AREA

---

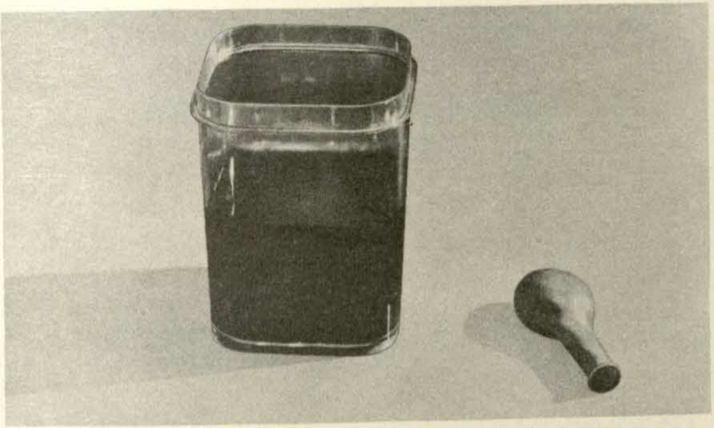
13 But suppose the sea and the mountains were real.
Would the sea have area only, or would it have volume as well?

12
MAP

---

14 Look at the map again.
The map has area, but it shows things that have volume, such as
(*a*) s – –, and
(*b*) m – – – – – – – – .

13
IT WOULD
HAVE VOLUME,
BUT ITS
SURFACE
COULD BE AN
AREA



15 Look at the picture.
The balloon has some volume.
The plastic jar has more volume.
Is the plastic jar full?

14
(*a*) SEA
(*b*) MOUNTAINS

---

16 The liquid has volume. It does not fill the jar.
Is its volume the same as that of the whole jar?

15
NO

---

17 The jar would hold more liquid.
We call the full inside volume of a container its CAPACITY (pronounced *ca-pa-city*).
There is 1 pint of liquid in the jar.
Is the CAPACITY of the jar greater than 1 pint?
Yes or no?

16
NO

---

18 The full INSIDE volume of this jar is its --------- .

17
YES

---

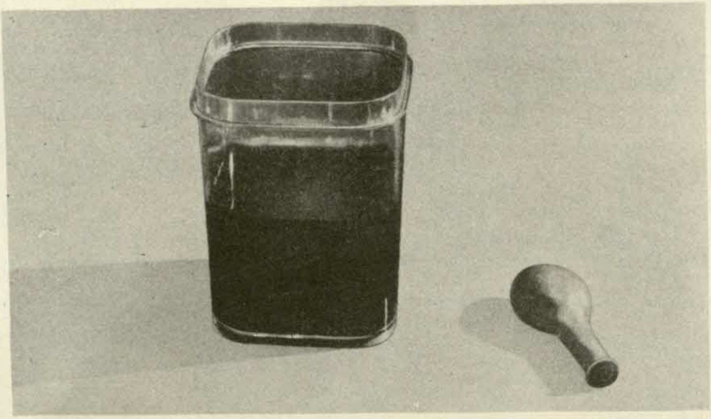
19 The balloon has c-------y to take in more air.

18
CAPACITY

---

20 If the balloon is blown up, will it be—
(*a*) larger?
(*b*) smaller?

19
CAPACITY



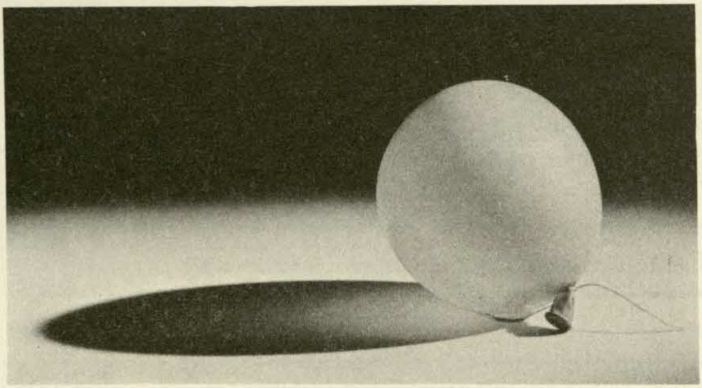
21 If the balloon is blown up, will it have more or less volume?

20
(*a*) LARGER

---

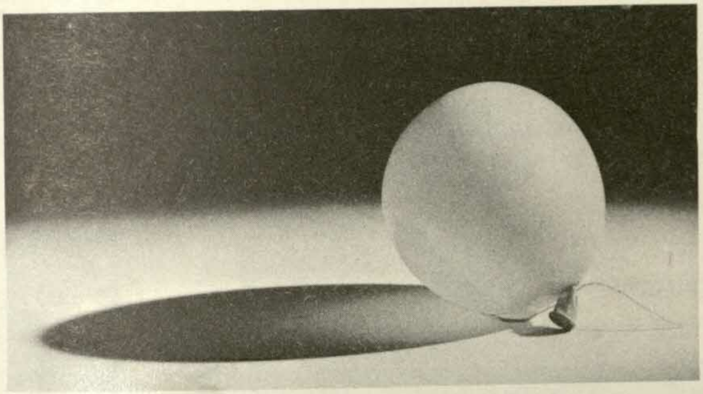
22 The c – – – – – – – – of a balloon means how much air it will hold.

21
MORE



23 The balloon has volume.
Its shadow has a – – – .

22
CAPACITY



24 The balloon is full of air. So the volume of air inside the balloon would be the balloon's --------- .

23
AREA

---

25 We must remember that the balloon has volume and it also has area. This is because the rubber *surface* of the balloon has an area. What about the shadow? Has it—

volume?
area and volume?
area only?

24
CAPACITY

---

26 If we burst the balloon and stretched out the skin tight and flat, we could measure it from side to side and front to back. We could not measure how deep it was.
Is this last sentence true or false?

25
AREA ONLY

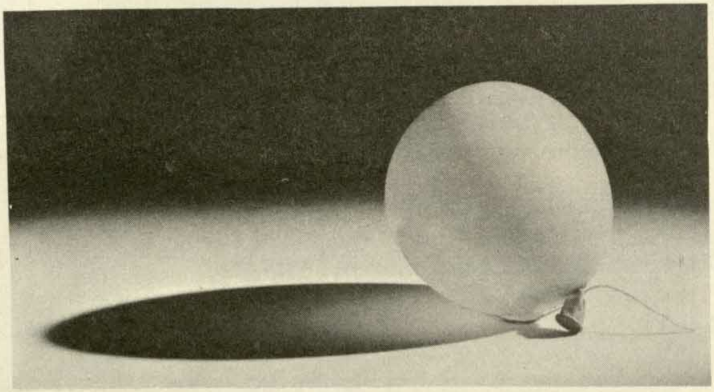
Let us revise.

Such objects as cups, balloons, oceans, and buildings, have volume.
Such things as shadows, flat surfaces, reflections, and the surface of the sea, have only area.

Volume can be measured three ways.
Area can be measured two ways.

Now answer this question in your notebook.

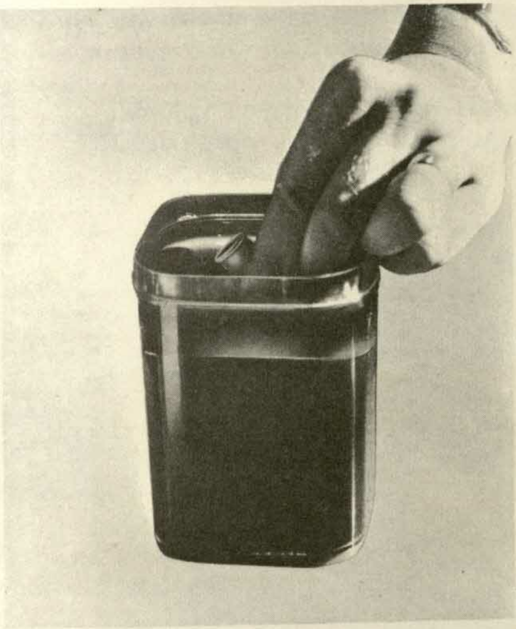
26
TRUE



27 In the picture above:
(*a*) name one object having volume and area;
(*b*) name something only measurable in area.

28 Object A measures:
I inch side to side;
I inch back to front;
I inch top to bottom.
Object B measures:
I foot side to side;
I foot back to front;
and has no top to bottom measurement.
Which object has volume?

27
(*a*) THE BALLOO
(*b*) THE SHADO



29 First look carefully at the picture in frame 15.
Now look at the photograph above.
The balloon is in the liquid.
(*a*) Is the liquid higher than before?
(*b*) What has made it higher?
The v – – – – – – of air in the balloon.
Write down the word in your notebook.

28
A

---

30 If we blew the balloon up more and pressed it right into the liquid in the same way, would the liquid—
(*a*) rise higher?
(*b*) go lower?

29
(*a*) YES
(*b*) VOLUME

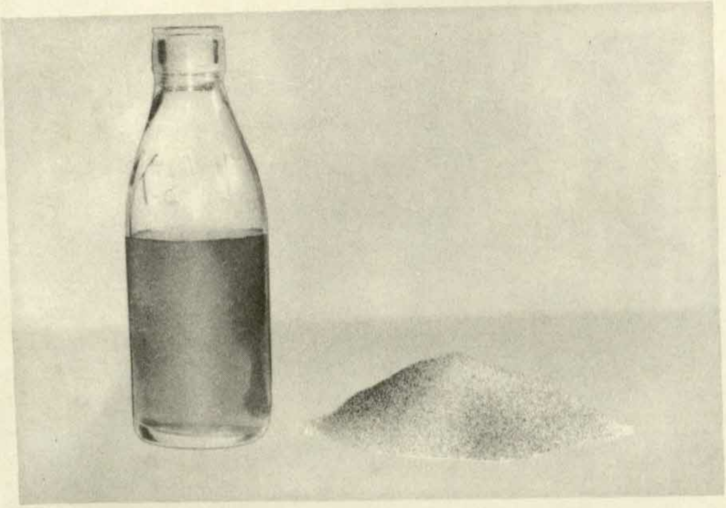
---

31 Did the liquid rise in the jar—
(*a*) because it was hot?
(*b*) because the balloon had volume?
(*c*) because the balloon was flat?

30
(*a*) RISE HIGHER

32 Remember that there is a difference between the volume an article takes up and its capacity.

For example; a milk bottle will always have a volume outside and a capacity inside. It measures more outside than it does inside.



Is the bottle in the picture filled to capacity?

31
(*b*) BECAUSE THE BALLOON HAD VOLUME

---

33 It is a pint bottle, so it has in it—
about 1 pint?
just over $\frac{1}{2}$ pint?

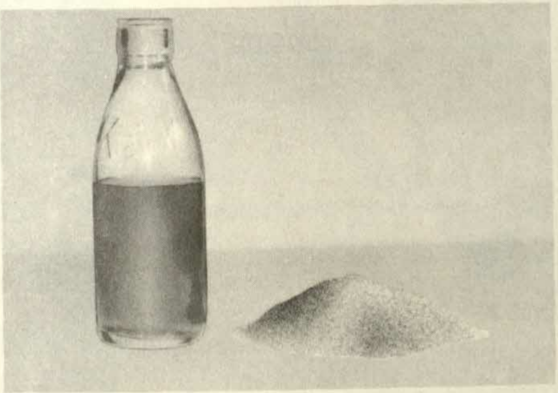
32
NO

---

34 We could say that in the bottle there is just over $\frac{1}{2}$ pint volume of liquid.
If we put in the sand instead, it would come to exactly the same height as the liquid.
What volume of sand is there—
1 pint?
just over $\frac{1}{2}$ pint?

33
JUST OVER $\frac{1}{2}$ PINT

35 We measure liquid volume in pints and quarts and gallons, but it can be measured in other ways. The same is true of sand—or sugar.



What is the capacity of this bottle to hold sand? (How much sand will it hold?)

34
JUST OVER
$\frac{1}{2}$ PINT

---

36 In what other way could we measure the sand, if we had some scales?

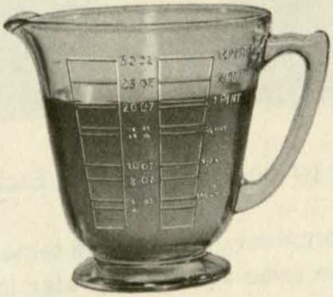
35
1 PINT

36
BY WEIGHT
IN POUNDS
OR OUNCES

37 Could we measure what weight of liquid is in the bottle?

---

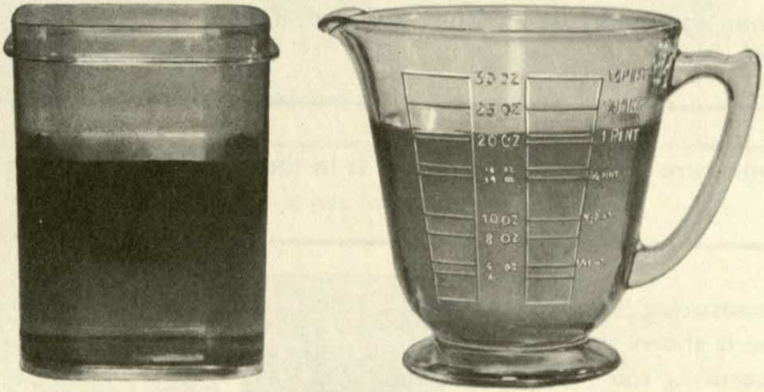
38 Here is a measuring jug.
You can see it shows two
ways of measuring the
volume of liquid (or sugar,
or flour, or sand).
One is by o - - - - -,
the other is by p - - ts.



37
YES

39 Remember, the contents of the jug are still liquid in volume. If we tipped the liquid all out it would spread over a large ----.

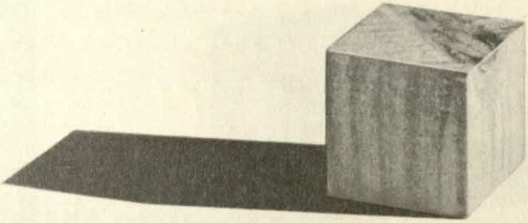
38
OUNCES (OZ.)
PINTS



40 Here are the jar and the jug again. Each has just I pint in it.

These two containers are not the same shape.
Have they the same volume of water in them?

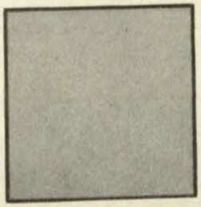
39
AREA



41 Let us revise again.
If an object has volume it can be measured three ways:
- side to side;
- top to bottom;
- and
- – – – – – to – – – – .

40
YES

42 The block in the picture in frame 41 measures 1″ along each side
So it measures:
- 1″ side to side,
- 1″ top to bottom,
- 1″ front to back.



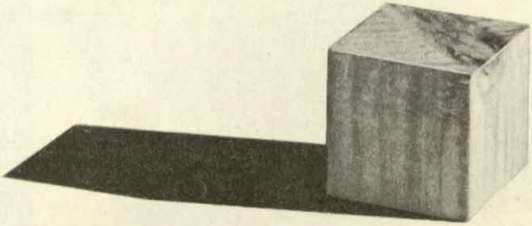
Look at the shaded face.
Remember the shaded face is just an AREA.

So it is 1″ × 1″ = 1——— inch.

41
FRONT TO BACK

43 But ALL the block has volume.
It measures 1″ × 1″ × 1″.
How many ways is it measured—
- 1 way?
- 2 ways?
- 3 ways?

42
SQUARE



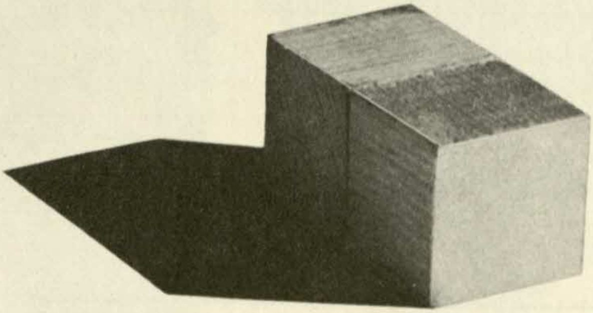
44 This block measures 1″ × 1″ × 1″.
It is a CUBE. (It has six faces. Count them.)
It can be measured 1″ side to side, 1″ back to front, 1″ top to bottom.
So it measures 1″ × 1″ × 1″ = – cubic inch?
(If you get the answer wrong, show it to your teacher at once.)

43
3 WAYS

---

45 The block is called a – – – – and measures – cubic inch.

44
1 CUBIC INCH



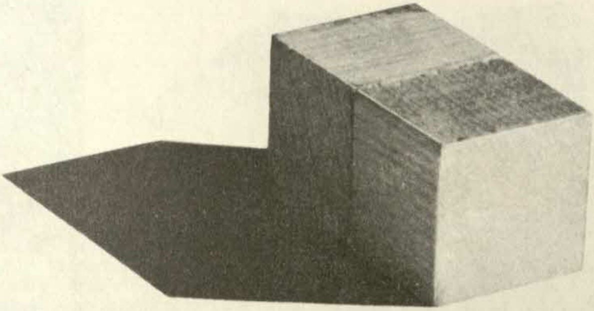
46 Here are two – – – – – .

45
CUBE
1

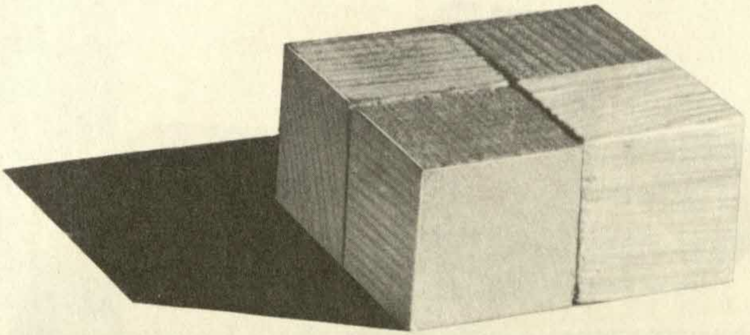
---

47 Each has sides of 1 inch. So each cube measures – cubic inch.

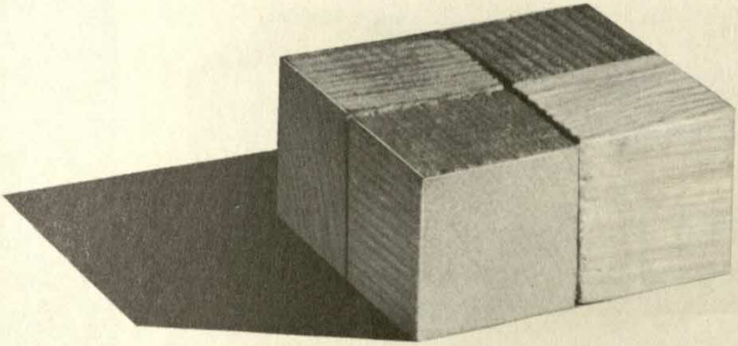
46
CUBES



48 So here we have altogether – cubic inches.	47 1 CUBIC INCH
49 If we separated the cubes, would there be— 2 cubic inches altogether? 1 cubic inch altogether? 4 cubic inches altogether?	48 2 CUBIC INCHES
50 If we put the cubes together again, what volume do they make up?	49 2 CUBIC INCHES
51 Here are the – cubic inches again. Each measures – cubic inch.	50 2 CUBIC INCHE
52 When placed side by side as in the photograph, they measure: 1″ top to bottom; 1″ front to back. How much from side to side?	51 2 1
53 So we say $1'' \times 1'' \times 2''$ is the cubic measurement. What does this work out to be?	52 2″
54 So our two cube inches put together work out to be: $2'' \times 1'' \times 1'' = -$?	53 2 CUBIC INCHES



55	How many cubic inches are there in the picture?	*54* 2 CUBIC INCHES
56	There are 4 ------ ------- .	*55* 4 CUBIC INCHES
57	If each side of each block measures 1″, then when they are placed together (as in the photograph in frame 55) what do they measure— (*a*) from side to side? (*b*) from front to back? (*c*) from top to bottom? Answer all three questions.	*56* 4 CUBIC INCHES, OR 4 CUBE INCHES
58	To find the cubic measurement of all 4 cubic inches together we work out: $2'' \times 2'' \times 1'' = -?$	*57* (*a*) 2″ (*b*) 2″ (*c*) 1″
59	Therefore we have - ----- ------- .	*58* 4 CUBIC INCHES
60	Look again at the picture in frame 55. What has both volume *and* area— (*a*) the shadow? (*b*) one of the blocks?	*59* 4 CUBIC INCHES
61	What has area only?	*60* (*b*) ONE OF THE BLOCKS



62 How many cubic inches are shown in the picture?

61
THE SHADOW

---

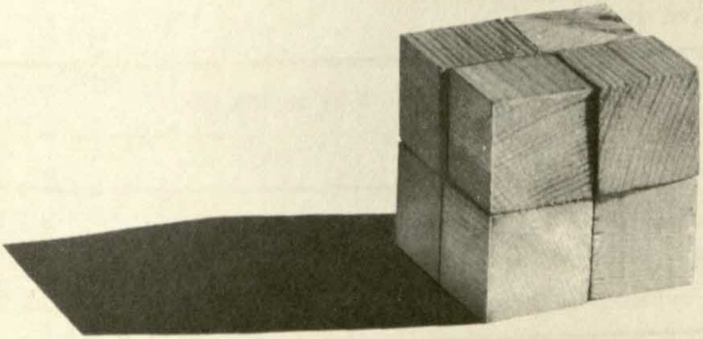
63 The volume of the blocks is 4 cubic inches:

side to side = 2″
front to back = 2″
top to bottom = 1″

How do we get the answer 4 cubic inches—

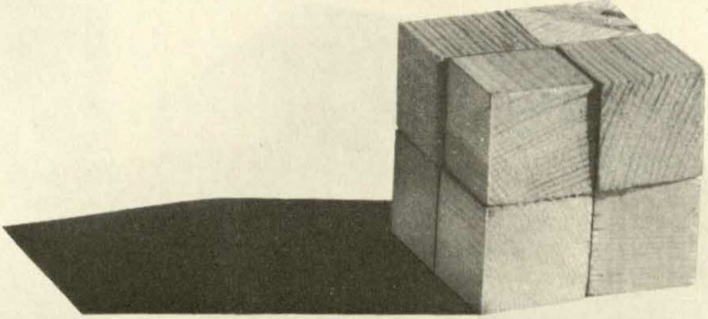
(*a*) by adding?
(*b*) by multiplying?

62
4 CUBIC INCHES

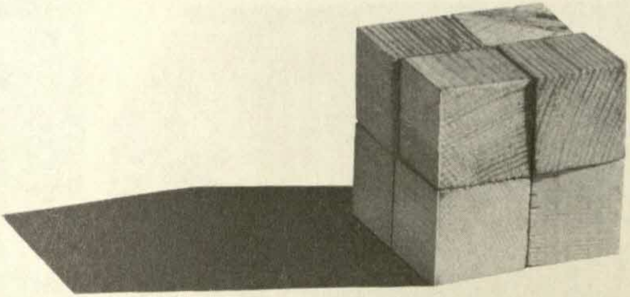


64 How many cube inches can you see in the picture?

63
(*b*) BY MULTIPLYING



65	How many cube inches are you unable to see?	*64* 7
66	How many cube inches are there altogether?	*65* 1
67	So here we have – cubic inches.	*66* 8
68	Each block is a 1″ cube. When put together as in the picture above, what do they measure— (*a*) from top to bottom? (*b*) from side to side? (*c*) from front to back? Answer all three questions.	*67* 8
69	So we work out the cubic measurements by saying 2″ × 2″ × 2″ = –?	*68* (*a*) 2″ (*b*) 2″ (*c*) 2″
70	One of the items in the photograph above can be measured in area only. What is it?	*69* 8 CUBIC INCHES
71	Here we have – – – – – – inches.	*70* THE SHADOW



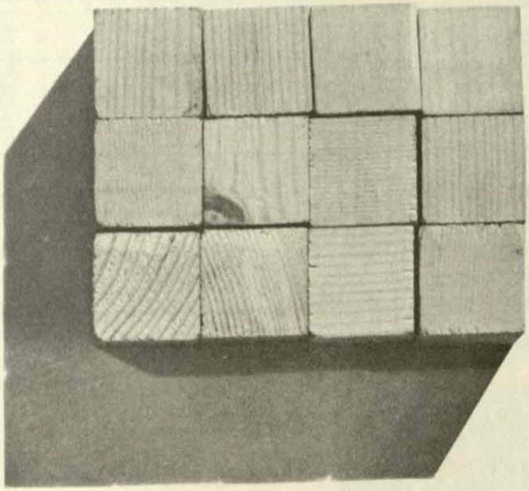
72 There are 8 cubes, each I cubic inch, which means we must have 8 cubic inches.
When they are together like this, how can we work out the answer in numbers?

71
8 CUBIC INCHES

---

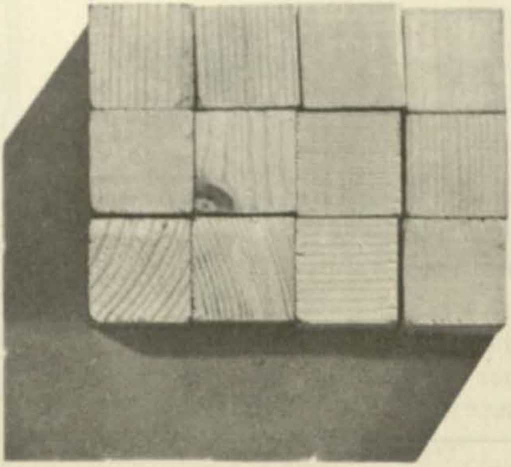
73 2″ × 2″ × 2″ gives us—what?

72
2″ × 2″ × 2″ =
8 CUBIC INCHES

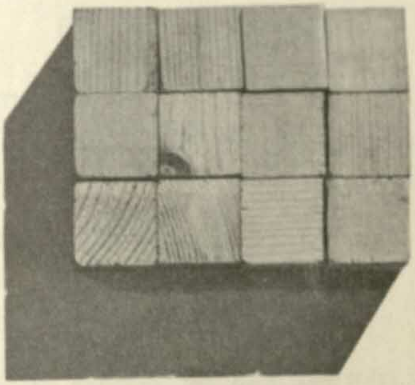


74 This photograph has been taken from immediately above.
In it you can see only I layer of inch cubes. (This means that the cubes are only I deep.)
How many cubic inches are there?

73
8 CUBIC INCHES



75	What do the cubes shown in the photograph measure— (*a*) from side to side? (*b*) from front to back? (*c*) from top to bottom (layer)? Answer all three questions.	74 12 CUBIC INCHES
76	How do we work out the answer— 4″ × 3″ × 1″ = –?	75 (*a*) 4 INCHES (*b*) 3 INCHES (*c*) 1 INCH
7	Because there is only 1 layer of blocks we have – – cubic inches.	76 12 CUBIC INCHES
78	The photograph has been taken from immediately above, but you are told that there are 2 layers of inch cubes. How many cubic inches can you see?	77 12 CUBIC INCHES
79	How many cubic inches are you unable to see?	78 12



80 There are 12 cubic inches you can see and 12 cubic inches you cannot see.
So altogether there are – cubic inches.

79
12

---

81 How can we work this out?
4×3× –

80
24

---

82 Work out the answer.

81
2

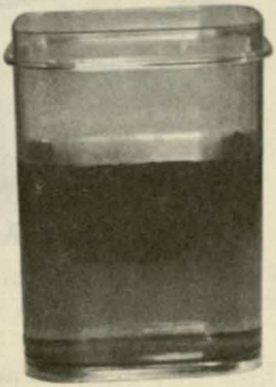
---

83 Because the blocks are 2 high we have – – cubic inches.

82
24 CUBIC INCHES

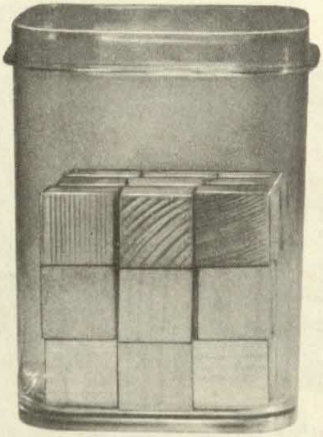
---

84 Look back at the picture in frame 40.
What volume of water is there in this jar?

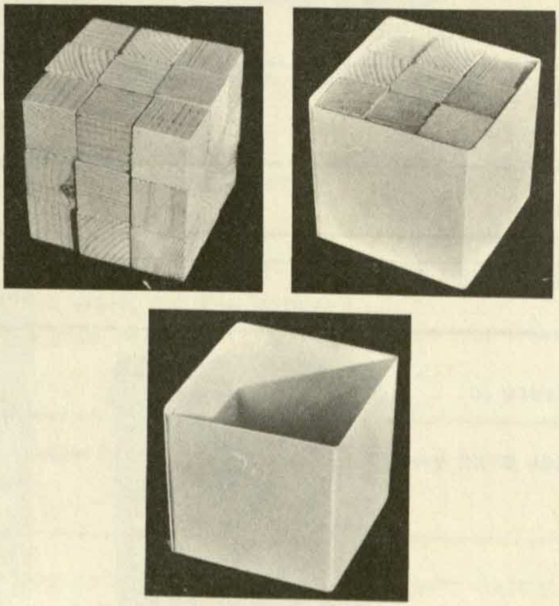


83
24 CUBIC INCHES

85 The cubes in the picture take up about 1 pint of volume. They stretch 3 deep from front to back. How many cubic inches are there?



84
I PINT



86 Look at these photographs. Now work out how you could CHECK the volume of the box if you had a ruler.

Now go on to the examples on the next page.

85
27 CUBIC INCHES

Problems

Try to answer the following questions before checking your answers (printed at the end of the book).

1 Draw a box which measures 2 inches by 3 inches at the top, and is 2 inches deep.
What is the volume of the box?

2 A CUBE has one side 10 inches long.
What is its volume?
(If you find this question hard to answer, try to draw a cube like this on a blackboard.)

3 A box of chocolates has a volume of 36 cubic inches.
If it is 6 inches long and 2 inches high, how wide must it be?

4 Suppose that we had a pint of milk, a square plastic box with transparent sides so that we could see in, and a ruler.
How could you work out what was the volume of a pint of milk in square inches instead of as a pint?

Answers to Tests on Area

Test 1	*Test 2*
A 6 square inches	A 12 square inches
B 20 square feet	B 24 square feet
C 24 square inches	C 16 square inches
D 42 square feet	D 23 square inches

Answers to Problems on Volume

1 A box 2 inches wide by 3 inches long and 2 inches deep measures:

$2 \times 3 \times 2 = 12$ cubic inches.

This means that the box would just hold 12 inch cubes and that its volume was 12 cubic inches.

2 If a cube has *one* side 10 inches long, then *all* its sides are 10 inches long and the cube would measure:

$10 \times 10 \times 10 = 1{,}000$ cubic inches.

3 The box of chocolates measures 36 cubic inches. 6 inches times 2 inches is 12 inches, so we need to divide the 36 by 12 to find the last side. The answer is 3 inches.

4 Pour the milk into the box. Measure the height, the width, and the depth of the milk. Then you can work out the volume. In this way you can measure a pint in cubic inches if you want to do so.